

Leptoquark on $P \rightarrow \ell^+ \nu$, FCNC and LFV

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Abstract

Motivated by the disagreement between the experimental data and lattice calculations on the decay constant of the D_s meson, we investigate leptoquark (LQ) contributions to the purely leptonic decays of a pseudoscalar (P). We concentrate on the LQs which only couple to the second-generation quarks before the electroweak symmetry breaking and we discuss in detail how flavor symmetry breaking effects are brought into the extension of the standard model after the spontaneous symmetry breaking. We show that the assumption of the hermiticity for the fermion mass matrices can not only reproduce the correct Cabibbo-Kobayashi-Maskawa and Maki-Nakagawa-Sakata matrices, but also reduce the number of independent flavor mixing matrices and lead to $V_f^R = V_f^L$ with $L(R)$ denoting the chirality of the f-type fermion. Accordingly, it is found that the decays $D_{s,d} \rightarrow \ell^+ \nu$, $B^+ \rightarrow \tau^+ \nu$ and $B_c \rightarrow \ell^+ \nu$ have a strong correlation in parameters. We predict that the decay constant of the B_c meson calculated by the lattice could be less than the experimental data by 23%. Intriguingly, the resultant upper limits of branching ratios for $D \rightarrow \mu^+ \mu^-$ and $\tau \rightarrow \mu(\pi^0, \eta, \eta', \rho, \omega)$ are found to be around 5.1×10^{-7} and $(2.6, 1.5, 0.6, 7.4, 4.8) \times 10^{-8}$, which are below and close to the current experimental upper bounds, respectively.

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As many puzzles such as matter-antimatter asymmetry, neutrino oscillations and dark matter etc are unsolved, it is clear that the standard model (SM) only describes parts of the universe and should be regarded as an effective theory at the electroweak scale. To explore the unknown parts, searching for new physics effects that do not belong to the SM becomes very important. However, since most measurements are eventually resulting from the SM, by naive speculation, the new effects should be small and difficult to be found out. Therefore, where we can uncover the new physics should be addressed in the first stage to look for new physics. Usually, the rare decays with the suppressed SM contributions are considered to be the good candidates. In addition, through precision measurements, finding a sizable deviation from the theoretical expectation provides another direction to search for the new effects.

Recently, via the observations of $D_s \rightarrow \ell^+ \nu$ decays, CLEO [1, 2] and BELLE [3] collaborations have measured the decay constant of D_s to be

$$\begin{aligned} f_{D_s} &= 274 \pm 10 \pm 5 \text{ MeV} \quad (\text{CLEO}), \\ f_{D_s} &= 275 \pm 16 \pm 12 \text{ MeV} \quad (\text{BELLE}), \end{aligned} \tag{1}$$

where the result by CLEO is the average of $\mu^+ \nu$ and $\tau^+ \nu$ modes, while the BELLE's one is only from $\mu^+ \nu$. By combining the radiative corrections from $D_s \rightarrow \gamma \ell \nu$, the average of the two data in Eq. (1) is [4]

$$f_{D_s} = 273 \pm 10 \text{ MeV}. \tag{2}$$

More information on the measurement from other experiments can be found in Ref. [4]. Furthermore, if we compare the measured value with the recent lattice calculation [5], given by

$$f_{D_s} = 241 \pm 3 \text{ MeV} \quad (\text{HPQCD+UKQCD}), \tag{3}$$

we see clearly that 3.2 standard deviations from data have been revealed in the purely leptonic D_s decays [4, 6]. That is, a correction of 10% to f_{D_s} is needed. Does the discrepancy indicate new physics or the defeat of the theory? Although the answer to the question is not conclusive yet, by following the new CLEO's result on the decay constant of D^+ [7]:

$$f_D = 205.8 \pm 8.5 \pm 2.5 \text{ MeV} \quad (\text{CLEO}), \tag{4}$$

which is in a good agreement with the lattice calculation [5]:

$$f_D = 208 \pm 4 \text{MeV} \quad (\text{Lattice}), \quad (5)$$

it seems to tell us that the lattice improvement may not be large enough to singly compensate the quantity that is more than 3 standard deviations in f_{D_s} .

Inspired by the above interesting measurements, the authors in Ref. [8] propose that new interactions associated with leptoquarks (LQs) might resolve the anomaly of f_{D_s} . However, the assumption adopted by Ref. [8] that the LQs only couple to the second-generation quarks seems to be oversimplified. It has been known that up-type and down-type quark mass matrices can not be diagonalized simultaneously. Therefore, if the LQ couples to up- and down-type quarks at the same time, after the spontaneous symmetry breaking (SSB), the flavor mixing matrices to diagonalize the quark mass matrices will be introduced so that intergenerational couplings in quarks become inevitably [9]. To generalize the approach of Ref. [8], in this paper, besides we discuss how flavor mixing effects influence the decays $P \rightarrow \ell^+ \nu$ and how the number of free parameters can be diminished, we also investigate the implications of LQ interactions on the processes with flavor changing neutral current (FCNC) and lepton flavor violation (LFV). We note that the effects of the charged Higgs with a large $\tan \beta$ in the ordinary two-Higgs-doublet models are destructive contributions to the SM [4, 10], more complicated multi-Higgs doublets are needed to get the enhancement [8]. In addition, other models such as R-parity violation in supersymmetric models might also provide the solution [11]. However, due to the parameters in different quark flavors having no correlation, the models have a less predictive power.

In order to examine the effects of a light LQ in a systematic way, the LQ model is built based on the gauge symmetries $SU(3)_C \times SU(2)_L \times U(1)_Y$. To simply display the role of the LQ on the low energy leptonic decays, FCNC and LFV, the LQ in this paper is limited to the $SU(2)_L$ singlet S_1 with the charge of $-1/3$. To avoid the proton decays, the LQ does not couple to diquarks. Indicated by the inconsistent results in the D_s leptonic decays, we consider that before the SSB, the LQ only couples to the second-generation quarks and the interactions in the weak eigenstates are given as [8, 12]

$$\begin{aligned} \mathcal{L}_{LQ} &= (\bar{E} g_L i \tau_2 P_R Q_2^c + \bar{\ell} g_R P_L c^c) S_1 + H.c., \\ &= (\bar{\ell} g_L P_R c^c - \bar{\nu}_\ell g_L P_R s^c) S_1 + \bar{\ell} g_R P_L c^c S_1 + H.c., \end{aligned} \quad (6)$$

where $g_{L(R)}$ denotes a 3-component effective coupling and is represented by $g_\alpha^T = (g_{\alpha e}, g_{\alpha \mu}, g_{\alpha \tau})$ with $\alpha = L$ and R , $Q_2^T = (c, s)$, $f^c = C\gamma^0 f^* = C\bar{f}^T$ ($C = i\gamma^2\gamma^0$) describes the anti-fermionic state, τ_2 is the 2nd Pauli matrix, $E^T = (\nu_\ell, \ell)$ with $\ell = e, \mu, \tau$, and $P_{L(R)} = (1 \mp \gamma_5)/2$. Since the flavor mixing effects are governed by the Yukawa sector, we write the sector as

$$\mathcal{L}_Y = -\bar{Q}_L Y_U U_R \tilde{H} - \bar{Q}_L Y_D D_R H - \bar{L} Y_L \ell_R H + H.c. \quad (7)$$

where H is the SM Higgs doublet and $\tilde{H} = i\tau_2 H^*$. Implicitly, the flavor indices are suppressed. In addition, it is known that the flavor changing effects in the SM only appear in processes related to the charged weak currents, while the weak interactions in weak eigenstates are expressed by

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{U}_L \gamma_\mu D_L + \bar{N}_L \gamma_\mu E_L) W^{+\mu} + H.c. \quad (8)$$

with g being the gauge coupling of $SU(2)_L$. After introducing the relevant pieces, in the following we discuss after the SSB how the flavor mixing effects are brought into the effective interactions and how they can be controlled through the notable patterns of mass matrices.

It has been known that Eq. (7) has $SU(3)_Q \times SU(3)_D \times SU(3)_U$ [16] flavor symmetries. As the new LQ interactions break the flavor symmetries, we have to be more careful to choose the convention. The one used in the SM is not suitable anymore for the new interacting terms. After the SSB, the masses of fermions are obtained by diagonalizing the Yukawa matrices denoted by Y_f with $f = U, D, E, N$. Although we do not display the mass matrix for neutrinos, due to the observations of the neutrino oscillation, we consider that neutrinos are massive particles. We will show that the induced effects such as the Maki-Nakagawa-Sakata (MNS) matrix [13] do not explicitly emerge after summing up the three neutrino species. To diagonalize the mass matrices of fermions, we introduce the unitary matrices through

$$f_\alpha^p = V_f^\alpha f_\alpha^w, \quad (9)$$

where $p(w)$ represents the physical (weak) state and α denotes the left or right-handness. Straightforwardly, Eq. (8) becomes

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{u}_L V_{CKM} \gamma_\mu d_L + \bar{\nu}_L V_{MNS} \gamma_\mu \ell_L) W^{+\mu} + H.c. \quad (10)$$

Here, $V_{CKM} = V_U^L V_D^{L\dagger}$ and $V_{MNS} = V_N^L V_E^{L\dagger}$ stand for the Cabibbo-Kobayashi-Maskawa (CKM) [14] and MNS matrices, respectively. Clearly, besides the CKM matrix, if we regard the neutrinos as massive particles, we bring in a new mixing matrix for leptons. However, does V_{MNS} have any effects on the low energy leptonic decays? The answer to the question in the SM is obvious. Since the neutrinos in hadronic decays are regarded as missing particles and are not detected, when one calculates the decay rate for the process, it is needed to sum up all neutrino species and the squared amplitude is associated with $\sum_\nu (V_{MNS})_{\nu\ell} (V_{MNS}^\dagger)_{\ell\nu}$. With the unitarity property, the result becomes $\sum_\nu (V_{MNS}^\dagger)_{\ell\nu} (V_{MNS})_{\nu\ell} = 1$ so that the effects of V_{MNS} do not show up explicitly. In sum, V_{MNS} in Eq. (10) could be rotated away by redefining the neutrino fields, *i.e.* the neutrinos produced via weak interactions are not the mass states propagating in the vacuum. Will the nonrotated V_{MNS} appear in the LQ interactions? To answer the question, we need to do more analysis on the LQ sector.

With the introduced unitary matrices, Eq. (6) in terms of physical states is transformed as

$$\begin{aligned} \mathcal{L}_{LQ} = & \bar{\ell}_L \tilde{g}_L V_{U2}^{LT} u_L^c S_1 - \bar{\nu} V_{MNS} \tilde{g}_L V_{D2}^{LT} d_L^c S_1 \\ & + \bar{\ell}_R \tilde{g}_R V_{U2}^{RT} u_R^c S_1 + H.c. \end{aligned} \quad (11)$$

where V_{U2}^α , V_{D2}^L and \tilde{g}_α are 3-component columns, represented by $V_{U2}^{\alpha T} = (V_{U12}^\alpha, V_{U22}^\alpha, V_{U32}^\alpha)$, $V_{D2}^{LT} = (V_{D12}^L, V_{D22}^L, V_{D32}^L)$ with $V_D^L = V_{CKM}^\dagger V_U^L$ and $\tilde{g}_\alpha = V_\ell^{L\dagger} g_\alpha$, respectively. Clearly, we see that V_{MNS} appears in Eq. (11). Nevertheless, like the SM, the explicit form of V_{MNS} can be rotated away by transforming the physical neutrino states to flavor states. Meanwhile, unlike the case in the SM where ν_ℓ in a process is always associated with the corresponding charged lepton ℓ , in the LQ model, for each charged lepton inevitably we have to consider all possible neutrino flavors.

Using Eqs. (10) and (11) with removing V_{MNS} , the effective Hamiltonian for $P \rightarrow \ell^+ \nu$

related decays are found to be

$$\begin{aligned}
\mathcal{H}(u_k \rightarrow d_i \ell_\ell^+ \nu_j) = & \frac{4G_F}{\sqrt{2}} (V^\dagger)_{ik} \delta_{\ell j} \bar{d}_i \gamma_\mu P_L u_k \bar{\nu}_j \gamma^\mu P_L \ell_\ell \\
& + \frac{(C_{dv}^{L\dagger})_{ji} (C_{ul}^L)_{k\ell}}{2m_{LQ}^2} \bar{d}_i \gamma_\mu P_L u_k \bar{\nu}_j \gamma^\mu P_L \ell_\ell \\
& - \frac{(C_{dv}^{L\dagger})_{ji} (C_{ul}^R)_{k\ell}}{2m_{LQ}^2} \bar{d}_i P_R u_k \bar{\nu}_j P_R \ell_\ell \\
& + \frac{(C_{dv}^{L\dagger})_{ji} (C_{ul}^R)_{k\ell}}{16m_{LQ}^2} \bar{d}_i \sigma_{\mu\nu} u_k \bar{\nu}_j \sigma^{\mu\nu} P_R \ell_\ell + H.c.
\end{aligned} \tag{12}$$

where we have used V as V_{CKM} , the indices i, j, k and ℓ denote the possible flavors,

$$C_{ul}^L = V_{U2}^{L*} \tilde{g}_L^\dagger, \quad C_{ul}^R = V_{U2}^{R*} \tilde{g}_R^\dagger, \quad C_{dv}^L = V_{D2}^{L*} \tilde{g}_L^\dagger \tag{13}$$

are 3×3 matrices, and $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$. Although tensor-type interactions could be generated in Eq. (12), since they cannot contribute to two-body leptonic decays, hereafter we will not discuss them further. Therefore, there are two main types of four-fermion operators in Eq. (12), one is $(V - A) \times (V - A)$, which is the same as the SM, and the other is $(S \pm P) \times (S \pm P)$. For $P \rightarrow \ell^+ \nu$ decays, we will see that the former will lead to the helicity suppression, whereas the latter does not. On the contrary, for $D \rightarrow \bar{K} \ell^+ \nu$ decays where the lattice calculations have been consistent with the experimental data, the latter has the helicity suppression whereas the former does not. Consequently, $D \rightarrow \bar{K} \ell^+ \nu$ will directly give strict constraints on the parameters $\tilde{g}_{L\ell}$. Since the new physics effects are considered perturbatively, if we only keep the leading effects and neglect the higher order in \tilde{g}_α , the partial decay rate for $P \rightarrow \ell^+ \nu$ is found to be

$$\begin{aligned}
\Gamma(P \rightarrow \ell^+ \nu) = & \Gamma^{SM} (1 + X_\ell^{UD} + Y_\ell^{UD}), \\
X_\ell^{UD} \approx & \frac{\sqrt{2}}{4G_F m_{LQ}^2} \frac{1}{|V_{UD}|^2} \text{Re} \left[V_{UD}^* (C_{ul}^{L\dagger})_{\ell U} (C_{dv}^L)_{D\ell} \right], \\
Y_\ell^{UD} \approx & \frac{\sqrt{2}}{4G_F m_{LQ}^2} \frac{m_P^0}{m_\ell |V_{UD}|^2} \text{Re} \left[V_{UD}^* (C_{ul}^{R\dagger})_{\ell U} (C_{dv}^L)_{D\ell} \right]
\end{aligned} \tag{14}$$

with $m_P^0 = m_P^2/(m_U + m_D)$ and

$$\Gamma^{SM} = \frac{G_F^2}{8\pi} |V_{UD}|^2 f_P^2 m_\ell^2 m_P \left(1 - \frac{m_\ell^2}{m_P^2} \right)^2, \tag{15}$$

where the decay constant f_P is defined by

$$\begin{aligned}
\langle 0 | \bar{D} \gamma_\mu \gamma_5 U | P(p) \rangle &= i f_P p_\mu, \\
\langle 0 | \bar{D} \gamma_5 U | P(p) \rangle &= -i f_P \frac{m_P^2}{m_D + m_U}.
\end{aligned} \tag{16}$$

Since CP problem is not concerned in this paper, for a further simplification of our numerical analysis, the weak phases will be tuned to zero. Then, X_ℓ^{UD} and Y_ℓ^{UD} can be shortened as

$$\begin{aligned} X_\ell^{UD} &\approx \frac{\sqrt{2}}{4G_F m_{LQ}^2} \frac{1}{V_{UD}} (C_{ul}^{L\dagger})_{\ell U} (C_{dv}^L)_{D\ell}, \\ Y_\ell^{UD} &\approx \frac{\sqrt{2}}{4G_F m_{LQ}^2} \frac{m_P^0}{m_\ell V_{UD}} (C_{ul}^{R\dagger})_{\ell U} (C_{dv}^L)_{D\ell}. \end{aligned} \quad (17)$$

Clearly, X_ℓ^{UD} and Y_ℓ^{UD} are associated with $|\tilde{g}_{L\ell}|^2$ and $\tilde{g}_{L\ell}^* \tilde{g}_{R\ell}$, respectively. We note that the capital symbol of $U(D)$ denotes the up (down)-type quark in a specific decay. For instance, X_ℓ^{cs} and X_ℓ^{ud} are for D_s and π decays, respectively.

Before doing the numerical analysis, we need to know how many free parameters are involved in the model and how to reduce the number of parameters. From Eq. (6), it is obvious that six free parameters from g_L and g_R are introduced in the original LQ model. These parameters are associated with the flavors of the charged leptons. After the SSB, due to the misalignment between mass and interaction states, we have to bring the new unitary matrices V_f^α to diagonalize the Yukawa matrices. Except that $V_{CKM} = V_U^L V_D^{L\dagger}$ is known by fitting the data, the elements in the unitary matrices are usually regarded as free parameters. In general, there is no any relationship among the flavor mixing matrices. Nevertheless, by utilizing the experimental data, we can obtain some clues to sense the information on mixing matrices. It is known that the determination of the flavor mixing matrices V_f^α is governed by the detailed patterns of the mass matrices. According to V_{CKM} being approximately a unity matrix, people have found that the quark mass matrices are very likely aligned and have the relationship of $\mathcal{M}_D = \mathcal{M}_U + \Delta(\lambda^2)$ with $\mathcal{M}_{U(D)} = M_{U(D)}/m_{t(b)}$ [17, 18, 19]. In other words, the structure of V_U^α should be similar to V_D^α . Furthermore, it has been shown that a simple pattern of the mass matrix, proposed by Ref. [20] with

$$\begin{aligned} M_f &= P_f^\dagger \bar{M}_f P_f \text{ with } \bar{M}_f = \begin{pmatrix} 0 & A_f & 0 \\ A_f & D_f & B_f \\ 0 & B_f & C_f \end{pmatrix}, \\ P_f &= (e^{i\theta_{f1}}, e^{i\theta_{f2}}, e^{i\theta_{f3}}), \end{aligned} \quad (18)$$

could lead to reasonable structures for the mixing angles and CP violating phase in the CKM matrix just in terms of the quark masses. Using the current accuracy of data, the mass patterns of Eq. (18) have been reanalyzed and applied to lepton masses by the authors

in Ref. [15]. It is found that the elements of V_{CKM} can satisfy with current accuracy of data and the component of $(V_{MNS})_{13}$ can be consistent with present experimental constraint as well. Although the phenomenological patterns may not be the general form, due to the support of experiments, the resultant flavor mixing matrices could be taken as a clue to the true mass matrices.

Inspired by the fascinating mass matrices and their results, we speculate that to avoid the restricted patterns shown in Eq. (18), the mass matrices could be extended to those which not only own the main character of Eq. (18) but also provide the relationship between V_f^R and V_f^L . Accordingly, we find that the criterion to get a more general property of Eq. (18) could be established if the mass matrices are hermitian. It is worth mentioning that the hermitian mass matrices could be naturally realized in gauge models such as left-right symmetric models [21]. Furthermore, the hermiticity is helpful to solve the CP problem in models with supersymmetry (SUSY) [22], which has an important implication on CP violation in Hyperon decays [23]. With the hermiticity, we obtain the results $V_f^L = V_f^R \equiv V_f$. Via $V_U = VV_D$ from the definition of the CKM matrix, intriguingly the number of independent flavor mixing matrices in the quark sector could be reduced to one and the unknown flavor mixing matrix is chosen to be V_D for our following analysis.

After setting up the model and the associated parameters, subsequently we study the constraints on the free parameters and their relative implications. Firstly, we discuss the limits of $D \rightarrow \bar{K}\ell^+\nu$. As mentioned early, the effects of \tilde{g}_R for $D \rightarrow \bar{K}\ell^+\nu$ are helicity suppressed. Here we only display the constraints on \tilde{g}_L . By the effective Hamiltonian of Eq. (12), the transition matrix element for $D \rightarrow \bar{K}\ell^+\nu$ can be written as

$$\begin{aligned} \mathcal{M}(D \rightarrow \bar{K}\ell^+\nu)_{SM+LQ} = & -\frac{G_F}{\sqrt{2}}V_{cs}^* \sum_j \left(\delta_{j\ell} + \frac{\sqrt{2}}{8G_F m_{LQ}^2} \frac{(C_{ul}^{L\dagger})_{\ell c} (C_{dv}^L)_{sj}}{V_{cs}} \right) \\ & \times \langle K | \bar{s} \gamma_\mu c | D \rangle \bar{\nu}_j \gamma^\mu (1 - \gamma_5) \ell_\ell, \end{aligned} \quad (19)$$

where the sum is to include all neutrino species and the $D \rightarrow \bar{K}$ form factors can be parametrized by

$$\langle \bar{K} | \bar{s} \gamma_\mu c | D \rangle = f_+(q^2) \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) + f_0(q^2) \frac{P \cdot q}{q^2} q_\mu. \quad (20)$$

If the effects of the 2nd order in \tilde{g}_L are neglected, a simple expression for $D \rightarrow \bar{K}\ell^+\nu$ is given by

$$\mathcal{B}(D \rightarrow \bar{K}\ell^+\nu)_{Exp} = (1 + X_\ell^{cs}) \mathcal{B}(D \rightarrow K\bar{\ell}\nu)_{SM}. \quad (21)$$

With $V_U = VV_D$, the effective coupling $(C_{ul}^{L\dagger})_{\ell c}(C_{dv}^L)_{s\ell}$ could be expressed as

$$(C_{ul}^{L\dagger})_{\ell c}(C_{dv}^L)_{s\ell} = (V_{cd}V_{D12} + V_{cs}V_{D22} + V_{cb}V_{D32})V_{D22}^*|\tilde{g}|_{L\ell}^2. \quad (22)$$

Since the off-diagonal elements of V_D represent the flavor symmetry breaking effects, according to Eq. (18), the diagonal elements of V_D are roughly order of unity while the off-diagonal elements are order of $\sqrt{m_i/m_j}$ with $j > i$ [19, 20, 24]. As a result, X_ℓ^{cs} could be written as

$$X_\ell^{cs} \approx 2\frac{m_W^2}{m_{LQ}^2} \begin{cases} |\tilde{g}_{Le}|^2/g^2 & \text{for } \ell = e, \\ |\tilde{g}_{L\mu}|^2/g^2 & \text{for } \ell = \mu \end{cases} \quad (23)$$

where $G_F/\sqrt{2} = g^2/8m_W^2$ has been used. From the data [25] and the recent unquenched lattice calculation [27], given by

$$\begin{aligned} \Gamma(D^0 \rightarrow K^- \ell^+ \nu)_{Exp} &= (8.17 \pm 0.48) \times 10^{-2} ps^{-1} & (\text{PDG}), \\ \Gamma(D^0 \rightarrow K^- \ell^+ \nu)_{Latt} &= (9.2 \pm 0.7 \pm 1.8 \pm 0.2) \times 10^{-2} ps^{-1} & (\text{Lattice}), \end{aligned} \quad (24)$$

obviously the theoretical calculation is consistent with the experimental value, *i.e.* we can set $\tilde{g}_{L\ell}$ as small as possible. In order to sense the order of magnitude of the parameters, we require that new physics effects are only less than $1\sigma_{Exp}$, *i.e.*

$$\frac{\Gamma_{Exp} - \Gamma_{Latt}}{\Gamma_{Latt}} = X_\ell^{cs} < 8\%. \quad (25)$$

Using $g \approx 0.67$ and $m_W \approx 80$ GeV, we get

$$\left(\frac{\tilde{g}_{L\ell'}}{m_{LQ}}\right)^2 < 2.8 \times 10^{-6} \text{ GeV}^{-2} \quad (26)$$

with $\ell' = e, \mu$.

Now, we study the LQ effects on $D_s \rightarrow \ell^+ \nu$ decays where the disagreement between theory and experiment shows up. In terms of the previous analysis, although the LQ only couples to the second-generation quarks, through the flavor mixing matrices, the LQ could also couple to the quarks of the first and third generations. Therefore, besides $D_s \rightarrow \ell^+ \nu$ decays, we can also study the processes $D_d \rightarrow \ell^+ \nu$ and $B_u \rightarrow \tau^+ \nu$, in which the involving parameters are correlated each other. Taking $V_{cs} \approx 1$, $V_{cd} = -\lambda \simeq 0.22$, $V_{D22} \approx 1$ and neglecting the subleading terms, from Eq. (17) the effects of LQ to $D_s \rightarrow \ell^+ \nu$, $D_d \rightarrow \ell'^+ \nu$

and $B_u \rightarrow \tau^+ \nu$ can be simplified to be

$$\begin{aligned}
X_\ell^{cs} &\approx 2 \frac{m_W^2}{m_{LQ}^2} \frac{\tilde{g}_{L\ell}^2}{g^2}, & Y_\ell^{cs} &\approx 2 \frac{m_W^2}{m_{LQ}^2} \frac{m_{D_s}^0}{m_\ell} \frac{\tilde{g}_{L\ell}^* \tilde{g}_{R\ell}}{g^2}, \\
X_{\ell'}^{cd} &\approx \frac{V_{D12}^*}{-\lambda} X_{\ell'}^{cs}, & Y_{\ell'}^{cd} &\approx \frac{V_{D12}^*}{-\lambda} \frac{m_{D_d}^0}{m_{D_s}^0} Y_{\ell'}^{cs}, \\
X_\tau^{ub} &\approx (V_{D12} + \lambda) \frac{V_{D32}^*}{V_{ub}} X_\tau^{cs}, & Y_\tau^{ub} &\approx (V_{D12} + \lambda) \frac{V_{D32}^*}{V_{ub}} \frac{m_B^0}{m_{D_s}^0} Y_\tau^{cs},
\end{aligned} \tag{27}$$

respectively. Clearly, the parameters contributing to $D_s \rightarrow \ell^+ \nu$ will also affect the decays $D_d \rightarrow \ell'^+ \nu$ and $B_u \rightarrow \tau^+ \nu$. Moreover, since the decay rate for $P \rightarrow \ell^+ \nu$ is directly related to the decay constant of the P-meson, to display the new physics effects, we express the connection of the observed decay constant with the lattice calculation to be

$$f_P^{Exp} = f_P^{Latt} \sqrt{1 + X_\ell^{UD} + Y_\ell^{UD}} \approx f_P^{Latt} \left(1 + \frac{X_\ell^{UD} + Y_\ell^{UD}}{2} \right). \tag{28}$$

To explain the anomalous results occurred in $D_s \rightarrow (\mu^+, \tau^+) \nu$ shown in Eqs. (2) and (3), the new physics at least should enhance f_{D_s} by 10%, that is, $X_{\mu(\tau)}^{cs} + Y_{\mu(\tau)}^{cs}$ should be around 20%. Due to $X_\ell^{cs} < 8\%$, we see that the dominant contributions are from Y_ℓ^{cs} . For simplicity, we will ignore the effects of X_ℓ^{cs} and adopt $Y_\ell^{cs} \approx 0.2$.

For $Y_{\ell'}^{cd}$, now we have the ambiguity in sign of V_{D12} , denoted by $\text{Sign}[V_{D12}]$. To understand the sign, we can refer to the result of Eq. (18) in which $\text{Sign}[V_{D12}] < 0$ [15]. With $Y_\ell^{cs} = 0.2$, we get $Y_{\ell'}^{cd} = 0.18|V_{D12}|/\lambda$. Since the results of the data and the lattice result on f_D are consistent each other, to fit the data within 1σ , one can find that the value of $|V_{D12}|$ should be less than 0.57λ where if $f_D^{Latt} = 204$ MeV is used, which leads to $f_D \approx 214.7$ MeV. As for Y_τ^{ub} , $\text{Sign}[V_{D32}]$ is also ambiguous. Again, the sign could be chosen to be the same as that provided by Eq. (18) in which $\text{Sign}[V_{D32}] > 0$. Comparing with f_{D_s} and f_D , although the error of f_{B_u} calculated by the lattice [28] is somewhat larger, due to the large enhancements of $1/|V_{ub}| \sim 1/\lambda^4$ and $m_B^0/m_{D_s}^0$, $B_u \rightarrow \tau^+ \nu$ can still give a strict limit on V_{D32} . Using the averaged value of $V_{ub} = 3.9 \times 10^{-3}$ [4] and $f_B = 216$ MeV, the SM prediction on the branching ratio (BR) is $\mathcal{B}(B_u \rightarrow \tau^+ \nu) = 1.25 \times 10^{-4}$. Taking the data with 1σ error and $\mathcal{B}(B_u \rightarrow \tau^+ \nu) = 1.85 \times 10^{-4}$ as the upper bound, we obtain $V_{D32} < 0.043$. By combining the above analysis, the instant predictions are the BRs for $B_c \rightarrow \ell^+ \nu$ decays. Similar to Eq. (27), the LQ contributions to B_c decays could be written as

$$Y_\ell^{cb} = \frac{V_{U22} V_{D32}^*}{V_{cb}} \frac{m_{B_c}^0}{m_{D_s}^0} Y_\ell^{cs}. \tag{29}$$

Adopting $V_{U22} \approx 1$, $V_{D32} \approx 0.043$, $V_{cb} \approx 0.042$ and $Y_\ell^{cs} \approx 0.2$, we immediately find $Y_\ell^{cb} \approx 0.49$. In other words, we predict that the calculation of the lattice on f_{B_c} could have $\sim 23\%$ below the observation of the experiment. According to above analysis, we see clearly that even in the restricted case, where the fermion mass matrices are hermitian, the explanation of the D_s puzzle in terms of the LQ remains viable despite constraints from other flavor processes.

With the constraints on the parameters of the LQ model, in the following we study the implications of the LQ effects on the decays associated with FCNC and LFV. Firstly, we discuss the $D \rightarrow \mu^+ \mu^-$ decay. It is known that due to the stronger Glashow-Iliopoulos-Maiani (GIM) mechanism [29], the short-distance contributions to $D \rightarrow \mu^+ \mu^-$ are highly suppressed in the SM [30] and long-distance effects are small [31]. The decay of $D \rightarrow \mu^+ \mu^-$ is definitely a good candidate to probe the new physics effects [32]. According to Eq. (11), we know that the dominant effective Hamiltonian for $c \rightarrow u \mu^+ \mu^-$ is from the left-right interference terms and can be written as

$$\begin{aligned} \mathcal{H}(c \rightarrow u \mu^+ \mu^-) = & -\frac{1}{2m_{LQ}^2} \left[(C_{ul}^L)_{c\mu} (C_{ul}^R)_{\mu u}^\dagger \bar{u} P_L c \bar{\mu} P_L \mu \right. \\ & \left. + (C_{ul}^R)_{c\mu} (C_{ul}^L)_{\mu u}^\dagger \bar{u} P_R c \bar{\mu} P_R \mu \right] + H.c. \end{aligned} \quad (30)$$

By combining Eqs. (13), (16), (27) and $V_U = V V_D$, the BR for $D \rightarrow \mu^+ \mu^-$ can be simplified to be

$$\begin{aligned} \mathcal{B}(D \rightarrow \mu^+ \mu^-) = & \tau_D \frac{m_D}{8\pi} \left(1 - \frac{4m_\mu^2}{m_D^2} \right)^{1/2} \left(\frac{G_F}{\sqrt{2}} \frac{m_D^0}{m_{D_s}^0} f_D m_\mu \right)^2 \\ & \times |Y_\mu^{cs}|^2 |V_{D12} + \lambda|^2. \end{aligned} \quad (31)$$

Using $Y_\mu^{cs} \approx 0.2$ and $V_{D12} \approx -0.57\lambda$, the values of BR with various values of f_D^{Latt} are presented in Table I. Interestingly, the LQ predictions satisfy and are close to the current experimental upper bound, given by $\mathcal{B}(D \rightarrow \mu^+ \mu^-)|_{Exp} < 5.3 \times 10^{-7}$ [26].

TABLE I: Upper limits of the LQ on $\mathcal{B}(D \rightarrow \mu^+ \mu^-)$ with various values of f_D^{Latt} . The upper bound of the current data is 5.3×10^{-7} [26].

$f_D^{\text{Latt}}(\text{MeV})$	204	206	208	210	212
BR	4.9×10^{-7}	5.0×10^{-7}	5.1×10^{-7}	5.2×10^{-7}	5.3×10^{-7}

The LQ interactions in Eq. (11) could also contribute to the lepton flavor violating processes. Since the constraints on the \tilde{g}_{Re} are more uncertain, we only pay attention to the decays $\tau \rightarrow \mu(P, V)$, in which the relevant effective Hamiltonian is

$$\mathcal{H}_{\tau \rightarrow \mu u \bar{u}} = -\frac{1}{2m_{LQ}^2} (C_{ul}^R)_{u\tau} (C_{ui}^R)_{\mu u}^\dagger \bar{u} \gamma^\mu P_R u \bar{\mu} \gamma_\mu P_R \tau + H.c. \quad (32)$$

For the light mesons, u represents the up-quark. By Eq. (13), the BRs for $\tau \rightarrow \mu(P, V)$ are given by

$$\begin{aligned} \mathcal{B}(\tau \rightarrow \mu P) &= \tau_\tau \frac{f_P^2 m_\tau^3}{2^{10} \pi} \left(1 - \frac{m_P^2}{m_\tau^2}\right)^2 \frac{|\tilde{g}_{R\tau}|^2 |\tilde{g}_{R\mu}|^2}{m_{LQ}^2 m_{LQ}^2} |V_{D12} + \lambda|^2, \\ \mathcal{B}(\tau \rightarrow \mu V) &= \tau_\tau \frac{f_V^2 m_\tau^3}{2^{10} \pi} \left(1 - \frac{m_V^2}{m_\tau^2}\right)^2 \left(1 + 2 \frac{m_V^2}{m_\tau^2}\right) \frac{|\tilde{g}_{R\tau}|^2 |\tilde{g}_{R\mu}|^2}{m_{LQ}^2 m_{LQ}^2} |V_{D12} + \lambda|^2, \end{aligned} \quad (33)$$

respectively. To calculate the modes associated with η and η' mesons, we employ the quark-flavor scheme in which η and η' physical states could be described by [33, 34]

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \quad (34)$$

with ϕ being the mixing angle, $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$. Accordingly, the decay constant of $\eta^{(\prime)}$ associated with $\bar{u}\gamma^\mu\gamma^5 u$ current is given by $f_{\eta^{(\prime)}} = \cos \phi (\sin \phi) f_{\eta_q}$. For numerical calculations, we have to know the direct bound on the free parameter $\tilde{g}_{R\tau}/m_{LQ}$. From Y_ℓ^{cs} of Eq. (27) and the result of Eq. (26), the information can be obtained immediately as

$$Y_\ell^{cs} \leq 1.3 \times 10^2 \frac{\tilde{g}_{R\ell}}{m_{LQ} m_\ell}.$$

With $Y_\ell^{cs} \approx 0.2$, the direct bound on $\tilde{g}_{R\ell}/m_{LQ}$ is found to be

$$\frac{\tilde{g}_{R\ell}}{m_{LQ}} \leq 1.6 \times 10^{-3} m_\ell. \quad (35)$$

By taking $\phi \approx 39^\circ$, $f_{\eta_q} \approx 140$ MeV [34], $f_\pi = 130$ MeV, $f_{\rho(\omega)} = 216(187)$ MeV, $V_{D12} \approx -0.57\lambda$ and the above resultant upper limits, the values of BRs for $\tau \rightarrow \mu(\pi^0, \eta, \eta', \rho^0, \omega)$ decays are displayed in Table II. We see that interestingly the contributions of the LQ to lepton flavor violating processes are below the current experimental upper bounds. In addition, the predictions on the decays $\tau \rightarrow \mu(\eta, \rho, \omega)$ are very close to the current upper bounds.

TABLE II: Upper limits of BRs from the current data [25, 35] and the LQ.

Mode	$\tau \rightarrow \mu\pi^0$	$\tau \rightarrow \mu\eta$	$\tau \rightarrow \mu\eta'$	$\tau \rightarrow \mu\rho^0$	$\tau \rightarrow \mu\omega$
Current limit	1.1×10^{-7}	6.5×10^{-8}	1.3×10^{-7}	2.0×10^{-7}	8.9×10^{-8}
This work	2.6×10^{-8}	1.5×10^{-8}	0.6×10^{-8}	7.4×10^{-8}	4.8×10^{-8}

In summary, to understand the inconsistency between the experimental data and lattice calculations in f_{D_s} , we have extended the SM to include the LQ interactions which involve only the second-generation quarks above the electroweak scale. After the SSB, the flavor mixing matrices introduced to diagonalize the mass matrices of quarks can make the LQ couple to the first and third generations. We have derived that if the mass matrices of fermions are hermitian in which the obtained CKM and MNS matrices can be consistent with data, besides having $V_f^R = V_f^L \equiv V_f$, the independent flavor mixing matrices are further reduced to one, say V_D . Accordingly, it is found that the effects of the LQ on the decays $D_{s,d} \rightarrow \ell^+\nu$, $B^+ \rightarrow \tau^+\nu$ and $B_c \rightarrow \ell^+\nu$ are correlated together. With the obtained constraints, we predict $f_{B_c}^{\text{Exp}} \approx 1.23 f_{B_c}^{\text{Latt}}$. Moreover, the upper limits of BRs for $D \rightarrow \mu^+\mu^-$ and $\tau \rightarrow \mu(\pi^0, \eta, \eta', \rho, \omega)$ are found to be around 5.1×10^{-7} and $(2.6, 1.5, 0.6, 7.4, 4.8) \times 10^{-8}$, respectively. Interestingly, all predicted values are below and close to the current experimental upper bounds.

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